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LETTER TO THE EDITOR

The axial gauge for gravity

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Abstract. Although fictitious field loops vanish when the axial gauge condition $n^\mu h_{\mu\nu}=0$ is imposed on the graviton field h, we find that the one-loop infinite counterterms in gravity nevertheless involve non-covariant terms such as $(n^\mu R_{\mu\nu} n^\nu)^2$; this is in contrast to the renormalisable Yang–Mills case where axis-dependent infinities do not arise.

Considerable work has been carried out for Yang–Mills theory in the axial gauge, even though a comprehensive determination of the two-loop effects is still lacking. On the other hand, the use of the axial gauge for gravity has barely been touched upon (Matsuki 1979). In this note we wish to point out the main features to be expected in such a gauge for gravitation, with particular reference to the one-loop infinities.

The popularity of the axial gauge (Delbourgo *et al* 1974, Kummer 1975) stems from three facts:

(a) Ward-Takahashi identities assume their simple, traditional forms,

(b) ghost field loops consequently decouple, (Frenkel 1976) and

(c) the infinite counter-terms are axis-independent (Konetschny and Kummer 1975) in any renormalisable theory such as QED or QCD. Thus the β-function in those theories is immediately related to wavefunction renormalisation.

While statements (a) and (b) apply just as well to gravity we shall discover that, because Einstein gravity is non-renormalisable, the counterterms depend explicitly on the fixed quantisation direction n. In expressing our results for gravitons we shall always make an analogy with vector mesons to emphasise the points of similarity and also the differences.

We shall take our graviton field to be defined by $g^{\mu\nu}=\eta^{\mu\nu}+Kh^{\mu\nu}$ in order to render the gravitational vertex functions simple. The axial gauge condition $n_\mu h^{\mu\nu}=0$, where n is a *constant* Minkowskian vector, can then be imposed by introducing a Lagrangian multiplier field b_ν and adding the gauge-fixing term

$$\mathscr{L}_b=\tfrac{1}{2}(n_\mu b_\nu+n_\nu b_\mu)h^{\mu\nu}. \tag{1}$$

Whereas in vector theories (where $\mathscr{L}_B=\boldsymbol{B}n\cdot\boldsymbol{A}$) no further gauge compensating terms arise, in gravity we must add to (1) the fictitious particle Lagrangian

$$\tfrac{1}{2}(n_\mu\bar{C}_\nu+n_\nu\bar{C}_\mu)(\partial_\lambda C^\mu\cdot g^{\lambda\nu}+\partial_\lambda C^\nu\cdot g^{\lambda\mu}-C^\lambda\partial_\lambda g^{\mu\nu})$$
$$=n_\mu\bar{C}_\nu\partial^\nu C^\mu+n_\nu\bar{C}_\mu\partial^\nu C^\mu+Kh^{\lambda\nu}(n_\mu\bar{C}_\nu\partial_\lambda C^\mu). \tag{2}$$

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Despite the presence of this term, Matsuki (1979) has shown on the basis of dimensional regularisation that any ghost loop will give zero anyhow because the ghost propagator just possesses $(k \cdot n)^{-r}$ singularities. *Thus the ghosts effectively decouple from gravitational processes.* Hence all one-loop and higher renormalisation effects come from intermediate graviton lines, and in view of gravitational gauge invariance one discovers that the graviton self energy is transverse, as Matsuki has anticipated. One might easily be led to assert that the gravitational counterterms are automatically generally covariant and have the form R^2, R^3, etc; after all this is what happens in Yang–Mills theory—the infinite counterterm is simply $\propto F_{\mu\nu}F^{\mu\nu}$. However, as we shall presently demonstrate, this assertion is false; although it is correct to claim that counterterms involve the curvature tensor, it is incorrect to assume that they are n-independent. This is the big difference between the axial gauge for gravity and for Yang–Mills and it is certainly tied to the fact that one theory cannot be renormalised while the other can.

In order to demonstrate these facts more fully we first construct the propagators from the bilinear terms

$$\mathscr{L}_2 = \tfrac{1}{2}(h_{\mu\nu,\lambda}h^{\mu\nu,\lambda} - h^{\mu}{}_{\mu,\lambda}h_{\nu}{}^{\nu,\lambda} + 2h^{\lambda}{}_{\lambda,\mu}h^{\nu\mu}{}_{,\nu} - 2h^{\nu}{}_{\mu,\nu}h^{\lambda\mu}{}_{,\lambda}) + \tfrac{1}{2}(n^{\mu}b^{\nu} + n^{\nu}b^{\mu})h_{\mu\nu} \tag{3}$$

and the vertex part from the trilinear term

$$\begin{aligned}\mathscr{L}_3 = {} & \tfrac{1}{2}(1 + Kh_{\kappa}{}^{\kappa})(h_{\mu\nu,\lambda}h^{\mu\nu,\lambda} - h^{\mu}{}_{\mu,\lambda}h_{\nu}{}^{\nu,\lambda} + 2h^{\lambda}{}_{\lambda,\mu}h^{\nu\mu}{}_{,\nu} - 2h^{\nu\mu}{}_{,\nu}h^{\lambda}{}_{\mu,\lambda}) \\ & - \tfrac{1}{2}Kh^{\mu\nu}(h_{\kappa\lambda,\mu}h^{\kappa\lambda}{}_{,\nu} - h^{\kappa}{}_{\kappa,\mu}h^{\lambda}{}_{\lambda,\nu} - 4h^{\kappa\lambda}{}_{,\mu}h_{\kappa\nu,\lambda} + 2h^{\kappa}{}_{\mu,\nu}h^{\lambda}{}_{\lambda,\kappa} + 2h^{\kappa}{}_{\kappa,\nu}h^{\lambda}{}_{\mu,\lambda} \\ & + 2h_{\mu\nu,\lambda}h^{\lambda\kappa}{}_{,\kappa} - 2h_{\mu\nu,\lambda}h_{\kappa}{}^{\kappa,\lambda} - 2h_{\mu\kappa,\lambda}h_{\nu}{}^{\lambda,\kappa} + 2h_{\mu\kappa,\lambda}h_{\nu}{}^{\kappa,\lambda}).\end{aligned} \tag{4}$$

Skipping over inessential kinematic details, one obtains from (3) the matrix propagator

$$\begin{aligned}&\begin{pmatrix} \langle h_{\kappa\lambda}, h_{\mu\nu}\rangle & \langle h_{\kappa\lambda}, b_{\nu}\rangle \\ \langle b_{\kappa}, h_{\mu\nu}\rangle & \langle b_{\kappa}, b_{\nu}\rangle \end{pmatrix} \\ &\quad = \begin{pmatrix} \Delta_{\kappa\lambda,\mu\nu}(p), & \Delta_{\kappa\lambda,\nu}(p) \\ \Delta_{\kappa,\mu\nu}(p), & \Delta_{\kappa,\nu}(p) \end{pmatrix} \\ &\quad = \begin{pmatrix} \dfrac{\mathscr{A}_{\kappa\mu\lambda\nu} + \mathscr{A}_{\kappa\nu\lambda\mu} + (1-l)^{-1}\mathscr{A}_{\kappa\lambda\,\mu\nu}}{2p^2}, & \dfrac{p_{\kappa}\eta_{\lambda\nu} + p_{\lambda}\eta_{\kappa\nu}}{2p\cdot n} - \dfrac{p_{\kappa}p_{\lambda}n_{\nu}}{2(p\cdot n)^2} \\ \dfrac{p_{\mu}\eta_{\kappa\nu} + p_{\nu}\eta_{\kappa\mu}}{2p\cdot n} - \dfrac{p_{\mu}p_{\nu}n_{\kappa}}{2(p\cdot n)^2}, & 0 \end{pmatrix}\end{aligned} \tag{5}$$

where

$$\mathscr{A}_{\mu\nu}(p) = -\eta_{\mu\nu} + \frac{p_{\mu}n_{\nu} + p_{\nu}n_{\mu}}{p\cdot n} - \frac{p_{\mu}p_{\nu}n^2}{(p\cdot n)^2}.$$

This is completely on a par with Yang–Mills where, instead, one meets

$$\begin{pmatrix} \langle A^a_{\kappa}, A^b_{\mu}\rangle & \langle A^a_{\kappa}, B^b\rangle \\ \langle B^a, A^b_{\mu}\rangle & \langle B^a, B^b\rangle \end{pmatrix} = \begin{pmatrix} \mathscr{A}_{\kappa\mu}/p^2, & p_{\kappa}/p\cdot n \\ p_{\mu}/p\cdot n, & 0 \end{pmatrix}^{\delta^{ab}}. \tag{5'}$$

The vertex part (incoming momenta p, q, r with associated indices as indicated) is read off from (4):

$$\Gamma_{\kappa\lambda,\mu\nu,\rho\sigma}(p,q,r)$$
$$=\sum \tfrac{1}{2}(r^2-\tfrac{1}{2}p\cdot q)(\eta_{\kappa\mu}\eta_{\lambda\nu}+\eta_{\kappa\nu}\eta_{\lambda\mu})\eta_{\rho\sigma}-\tfrac{1}{4}(p^2+q^2+r^2)\eta_{\kappa\lambda}\eta_{\mu\nu}\eta_{\rho\sigma}$$
$$-\tfrac{1}{8}(p^2+q^2+r^2)\sum \eta_{\rho\mu}\eta_{\kappa\sigma}\eta_{\nu\lambda}$$
$$+\sum \tfrac{1}{2}(r_\kappa r_\lambda+q_\kappa q_\lambda)\eta_{\mu\nu}\eta_{\rho\sigma}+\sum[\tfrac{1}{4}(q_\kappa r_\lambda+q_\lambda r_\kappa)-\tfrac{1}{2}p_\kappa p_\lambda](\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})$$
$$+\sum \tfrac{1}{4}(r_\nu q_\rho-2p_\nu p_\rho)\eta_{\mu\sigma}\eta_{\kappa\lambda}+\sum \tfrac{1}{4}(r_\nu r_\sigma-p_\nu q_\sigma)(\eta_{\kappa\mu}\eta_{\lambda\rho}+\eta_{\kappa\rho}\eta_{\lambda\mu}) \qquad (6)$$

(the summation being carried out over distinct permutations) and one verifies that it correctly satisfies

$$p^\kappa\Gamma_{\kappa\lambda,\mu\nu,\rho\sigma}(p,q,r)$$
$$=-\tfrac{1}{2}q_\lambda\Delta^{-1}{}_{\mu\nu,\rho\sigma}(r)-\tfrac{1}{2}r_\lambda\Delta^{-1}{}_{\mu\nu,\rho\delta}(q)$$
$$-\tfrac{1}{2}p_\kappa(\eta_{\lambda\sigma}\Delta^{-1}{}_{\kappa\rho,\mu\nu}(q)+\eta_{\lambda\rho}\Delta^{-1}{}_{\kappa\sigma,\mu\nu}(q)+\eta_{\lambda\nu}\Delta^{-1}{}_{\kappa\mu\rho\sigma}(r)+\eta_{\lambda\mu}\Delta^{-1}{}_{\kappa\nu\rho\sigma}(r)). \qquad (7)$$

Again this is similar to Yang–Mills,

$$p^\kappa\Gamma_{\kappa\mu\rho}(p,q,r)=\Delta^{-1}{}_{\mu\rho}(q)-\Delta^{-1}{}_{\mu\rho}(r). \qquad (7')$$

With (7) established, it is trivial to prove that the graviton self energy Π is transverse, $p^\kappa\Pi_{\kappa\lambda\kappa'\lambda'}(p)=0$ for any respectable regularisation, such as dimensional continuation. Matsuki's general argument is thus confirmed.

One may expand Π in terms of the transversal projectors $S_{\mu\nu}$, $T_{\mu\nu}$ introduced by Kummer (1975), or alternatively in terms of $S_{\mu\nu}$ and

$$(S+T)_{\mu\nu}=d_{\mu\nu}=\eta_{\mu\nu}-p_\mu p_\nu/p^2. \qquad (8)$$

We remind the reader that for temporal $n=(1;\underline{0})$, $S_{\mu 0}=0$ and $S_{ij}=-\delta_{ij}+p_ip_j/\underline{p}^2$. This then leads to five kinematic covariants†

$$\Pi_{\kappa\lambda\kappa'\lambda'}=\tfrac{1}{2}[S_{\kappa\kappa'}S_{\lambda\lambda'}+S_{\kappa\lambda'}S_{\lambda'\kappa}-2S_{\kappa\lambda}S_{\kappa'\lambda'}]\underline{p}^4A+S_{\kappa\lambda}S_{\kappa'\lambda'}\underline{p}^4B+[S_{\kappa\lambda}d_{\kappa'\lambda'}+d_{\kappa\lambda}S_{\kappa'\lambda'}]p^2\underline{p}^2C,$$
$$+\tfrac{1}{2}[d_{\kappa\kappa'}d_{\lambda\lambda'}+d_{\kappa\lambda'}d_{\lambda'\kappa}-2d_{\kappa\lambda}d_{\kappa'\lambda'}]p^4D+d_{\kappa\lambda}d_{\kappa'\lambda'}p^4E. \qquad (9)$$

The scalar invariants A to E may be extracted via five independent contractions $n^\kappa n^\lambda\Pi_{\kappa\lambda\kappa'\lambda'}n^{\kappa'}n^{\lambda'}$, $\Pi_\kappa{}^\kappa{}_{\kappa'\lambda'}n^{\kappa'}n^{\lambda'}$, $\Pi_\kappa{}^\kappa{}_{\kappa'}{}^{\kappa'}$, $n^\kappa\Pi_{\kappa\lambda\kappa'}{}^\lambda n^{\kappa'}$, $\Pi_{\kappa\lambda}{}^{\kappa\lambda}$. Now if the final result were generally covariant, only D and E would carry non-zero infinities and those could be related to the usual counterterms‡ ('t Hooft and Veltman 1974) $R_{\mu\nu}R^{\mu\nu}$ and R^2. However if A, B and C also possess infinities then they should be associated with further counterterms connected with curvature components orthogonal to n, namely

$$(n^\mu n^\nu R_{\mu\nu})^2,\ n^\mu n^\nu R_{\mu\nu}R \quad\text{and}\quad n^\mu R_{\mu\nu}R^{\nu\lambda}n_\lambda. \qquad (10)$$

To find out if such new counterterms are in fact present it suffices to calculate the scalar contraction

$$\Pi_\kappa{}^\kappa{}_{\kappa'}{}^{\kappa'}=p^4(-6D+9E)+12p^2\underline{p}^2C+\underline{p}^4(-2A+4B). \qquad (11)$$

† $\underline{p}^2$ signifies $[(p\cdot n)^2/n^2-p^2]$ for a more general choice of axis n, in (9).

‡ In Yang–Mills theory $\Pi_{\kappa\kappa'}=S_{\kappa\kappa'}\underline{p}^2\pi^N+d_{\kappa\kappa'}p^2\Pi^C$ is the analogue of (9). There only Π^C is infinite and is of course related to the wavefunction counterterm $(Z_3-1)F^2$.

The existence of infinities multiplying $p^2\underline{p}^2$ and $\underline{p}^4$ would signal axis-dependent counterterms. We have carried out this calculation and discovered that†

$$\Pi_{\kappa}{}^{\kappa}{}_{\kappa'}{}^{\kappa'}|_{\infty} = K^2 \ln(\Lambda^2/\mu^2)[\tfrac{9}{4}p^4 - \tfrac{19}{3}p^2\underline{p}^2 - \tfrac{88}{15}\underline{p}^4] \tag{12}$$

from which we conclude that *extra counterterms, beyond the generally covariant ones, will be needed for gravity in the axial gauge.* Presumably the n-dependence persists for higher order loops (Van Nieuwenhuizen and Wu 1977) with corresponding R^3, R^4, counterterms and ever-larger varieties of products involving R and n. Of course for pure gravity all these higher R^n corrections disappear on-shell.

Details of this work will be presented at greater length in a separate paper; where absorptive parts of Π will be given and the coefficients of all five R infinities will be exposed.

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† The method of computation is spelt out in Beven and Delbourgo (1978). The logarithms ultraviolet/infrared, infinity in (12) is construed as an $(n-4)^{-1}$ pole in dimensional regularisation.